Anisotropic Mesh Adaptation for the Minimization of the Ambrosio-Tortorelli Functional with Application to Quasi-static Brittle Fracture Propagation

Marco Artina
Mathematics, Technische Universität München, Germany

Francfort and Marigo presented in 1998 a model for quasi-static brittle fracture which requires the minimization of the Mumford-Shah functional, representing the energy of the system. The minimization of this functional represents a very challenging issue since it is non-smooth and non-convex. The numerical approximation of this problem can be issued via a Gamma-approximation on the energy functional proposed by Ambrosio and Tortorelli where a smooth indicator function identifies the fracture. Then, we resort to an adaptive finite element approach based on piecewise linear elements. Nevertheless, similarly to early work by Chambolle et al. but differently from recent approaches by Sli et al. where isotropic meshes are used, in this work we investigate how anisotropic meshes can lead to significant improvements in terms of the balance between accuracy and complexity. In fact, the employment of these grids allows us to shortly follow the propagation of the fracture by refining it only in a very thin neighborhood of the crack. Moreover, the main gain achievable is a relevant reduction of the number of elements needed to obtain with good confidence the expected behavior of the crack with respect to the techniques used in other works. In this talk, we first present the derivation of a novel anisotropic a posteriori error estimator driving the mesh adaptation for the approximation of the Ambrosio-Tortorelli model. Then, we provide several numerical results which corroborate the accuracy as well as the computational saving led by an anisotropic mesh adaptation procedure.